

STANDARD FOR MATHEMATICAL
PRACTICE 1:

**Make sense of problems and persevere
in solving them.**

*I can determine what the problem is asking me to do
and not give up until I've solved it.*

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

In short, mathematically proficient students:

- Interpret and make meaning of the problem to find a starting point.
- Analyze what is given in order to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their own progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, “Does this make sense?”
- Can understand various approaches to solutions.

National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010c



Make sense of problems and persevere in solving them.

Learning Progression

PRE-K	In Pre-K, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. With prompting and support from adults, students explain the meaning of a problem and look for ways to solve it. Students use concrete objects to help them conceptualize and solve problems. They may check their thinking by, with guidance asking, "Does this make sense?" or they may try another strategy.
KINDERGARTEN	In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
FIRST GRADE	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.
SECOND GRADE	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach.
THIRD GRADE	In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
FOURTH GRADE	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

FIFTH GRADE	In fifth grade, students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
SIXTH GRADE	In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
SEVENTH GRADE	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
EIGHTH GRADE	In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
HIGH SCHOOL	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Practice
1

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"I can determine what the problem is asking me to do and not give up until I've solved it."

Process

Process to make sense of problems

1. Read the problem out loud.
2. Identify and clarify each word that tells you what to do mathematically.
3. Explain the problem in your own words.
4. Explain how you will know you have solved the problem correctly.

Process to persevere in solving them

1. Make a plan for solving the problem.
2. Begin to solve the problem.
3. Each time you get stuck, identify where you got stuck.
4. Ask for help, as needed.
5. Keep working until you've solved the problem correctly.

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Reflection

To what degree can you determine what the problem is asking you to do and not give up until you've solved it?

1
never

2

3
sometimes

4

5
always

- ▶ What does "make sense of problems" mean?

- ▶ What does "persevere in solving them" mean?

- ▶ How do you determine what the problem is asking you to do?

- ▶ How do you not give up until you've solved the problem?

- ▶ How does "making sense of problems and persevering in solving them" help you?

Practice
1

Make sense of problems and persevere in solving them.

Pre-K

The Practice in Action: When presented with a problem that asks students to copy and extend patterns, pre-k students utilizing this practice repeat what information the problem has given them. Then, they explain what they are doing as they work through the problem. If they get stuck, they ask for help.

Model for your students how to build an AB color pattern using two colors of cubes. Then read the pattern together as you touch each cube and name its color (ex: red-blue-red-blue). Encourage students to copy your AB pattern with their own blocks. Read their pattern together and then ask them to extend their pattern by naming what color comes next.

TEACHER: Let's make a color pattern. I can use these red and blue cubes. Watch me make a pattern...red-blue-red-blue-red-blue. I made a pattern. Let's read it together...red-blue-red-blue-red-blue. What are we doing?

STUDENT: *Making a pattern.*

TEACHER: That's right! We are making a color pattern. We have two colors and they repeat, red then blue, over and over. Now you make the same pattern with two different colors. How can you get started?

STUDENT: *I like green, but I don't know how to do it. Can you help me?*

TEACHER: Of course! I'm glad you asked for help when you got stuck. You need two colors and you've chosen one. Let's put the green cube down first. Now pick another color.

STUDENT: *Pink!*

TEACHER: Great! Let's put the pink cube down next. Now remember our first pattern repeated...red-blue-red-blue-red-blue...This pattern has green-pink...what comes next?

STUDENT: *Green then pink then green then pink then green then pink...*

TEACHER: Fantastic! You made a pattern. You also made sense of this problem and persevered in solving it. That means you figured out what to do and didn't give up when you got stuck. How does making sense of the problem and persevering in solving it help you?

STUDENT: *It helps me know to make a pattern.*

For students who are developing language, it is crucial that teachers model how to ask questions, how to give answers, and how to explain their thinking in a variety of ways. Use these questions and the statements below to guide your modeling and discussions.

Questions to Foster Metacognition:

What is the problem asking you to do?

What are we going to do?

How can you get started?

How do you want to solve the problem?

What comes next?

What do you think?

Why do you think so?

Does this make sense?

What can you do if you get stuck?

How does making sense of the problem and persevering in solving it help you?

Ownership Statements:

The problem is asking me to _____.

We are going to _____.

I can start to solve the problem by _____.

When I am stuck, I can _____.

I solved the problem by _____.

Not giving up helps me because _____.

Making sense of problems and persevering in solving them helps me _____.

Practice
1

Make sense of problems and persevere in solving them.

Kindergarten

The Practice in Action: When presented with a problem that asks students to understand addition as putting together any number from 1 to 9 to find the number that makes 10 when added to the given number, kindergarteners utilizing this practice repeat what information the problem has given them. Then, they explain what the problem is asking them to do. If they get stuck, they use drawings, counters, or other tools and strategies to help them.

There are 5 red apples on the table. If you add 3 green apples, how many total apples are on the table.

TEACHER: What is the problem asking you to do?

STUDENT: *I have to add 5 red apples and 3 green apples to find out how many apples in all.*

TEACHER: How can you get started?

STUDENT *I can start with red and green counters to help me.*

TEACHER: What is your plan to solve it?

STUDENT *First, I will make a group of red and a group of green. Then I have to count them together. 1-2-3-4-5 red...1-2-3 green... Now I will count them all...1-2-3-4-5-6-7-8...There are eight in all.*

TEACHER: How does making sense of problems and persevering in solving them help you?

STUDENT *It helps me know what to do. I can make a plan and work until I have the answer.*

For students who are developing language, it is crucial that teachers model how to ask questions, how to give answers, and how to explain their thinking in a variety of ways. Use these questions and the statements below to guide your modeling and discussions.

Questions to Foster Metacognition:

What is the problem asking you to do?

How can you get started?

What is your plan to solve the problem?

What are some other ways you could solve the problem?

Does this make sense?

What can you do if you get stuck?

How do you know if you are stuck?

How do you know when you have solved the problem?

How does making sense of problems and persevering in solving them help you?

Ownership Statements:

The problem is asking me to _____.

I can start to solve the problem by _____.

When I am stuck, I can _____.

I solved the problem by _____.

I know when I have the answer because _____.

Not giving up helps me because _____.

If I can't solve the problem yet, I try again by _____.

Making sense of problems and persevering in solving them helps me _____.

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First Grade

The Practice in Action: When presented with a problem involving adding and subtracting within 20, first graders utilizing this practice explain what the problem is asking them to do and look for ways to solve it. If they get stuck, they use concrete objects, such as cubes and counters to conceptualize and solve the problem. As a way of checking their thinking, they ask themselves, “Does this make sense?”

What is 12 - 5?

TEACHER: What is the problem asking you to do?

STUDENT: *The problem is asking me to find the answer to twelve minus five.*

TEACHER: What is your plan to solve the problem?

STUDENT: *I can draw a group of twelve circles, X out five, and count how many are left.*

TEACHER: What can you do if you get stuck?

STUDENT: *If I get stuck, I can use counters instead of my drawing. The answer is seven, but I can check my work by adding five plus seven and make sure it equals twelve. Then I know I have the answer.*

TEACHER: How does making sense of the problem and persevering in solving it help you?

STUDENT: *I have to think about what the problem is, first. Then I can try different ways to solve it. I know my answer is right because twelve minus five is the same as taking five things away from a group of twelve and that's what I did.*

Questions to Foster Metacognition:

What is the problem asking you to do?

How can you get started?

What is your plan to solve the problem?

What are some other ways you could solve the problem?

Does this make sense?

What can you do if you get stuck?

How do you know if you are stuck?

How do you know when you have solved the problem?

How does making sense of problems and persevering in solving them help you?

Ownership Statements:

The problem is asking me to _____.

I can start to solve the problem by _____.

When I am stuck, I can _____.

I solved the problem by _____.

I know when I have the answer because _____.

Not giving up helps me because _____.

If I can't solve the problem yet, I try again by _____.

Making sense of problems and persevering in solving them helps me _____.

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Second Grade

The Practice in Action: When presented with a problem that asks students to use place value understanding and properties of operations to add and subtract, second graders utilizing this practice explain to themselves the meaning of the problem and discuss how they will solve it. If they are stuck or are struggling, they can use tools to check their answers. They check their thinking by asking themselves, “Does this make sense?”

Determine how many students are currently present, if there are 29 students on the playground and then 18 more students show up.

TEACHER: What is the problem asking you to do?

STUDENT: *The problem is asking me to figure out how many students are on the playground.*

TEACHER: How can you get started?

STUDENT: *I will start by saying what I know from the problem. I know that at first there were 29 students. Then 18 more students showed up. I know that the word ‘more’ means ‘add’. This problem is asking me to add 29 plus 18. So, I will add the two numbers together to find the answer.*

TEACHER: What is your plan to solve the problem?

STUDENT: *I will solve the problem by writing the two numbers and lining up the ten’s and one’s places. Then I add the ones together. Nine plus eight equals 17. 17 is one ten and seven ones. So, I can write seven in the one’s place and carry the one ten to the ten’s place. Then I add the ten’s place. Two tens plus one ten plus the one ten I carried equals four tens. So, I write four. The answer is 47.*

TEACHER: How does making sense of the problem and persevering in solving it help you?

STUDENT: *Making sense of problems and persevering in solving them means that I can’t just look at the numbers. I have to think about what the problem is asking and think about what I know. Then I have to think about how I will solve it. I have to go slow and make sure it makes sense. This helps me get my answers right.*

Questions to Foster Metacognition:

What is the problem asking you to do?

How can you get started?

What is your plan to solve the problem?

What are some other ways you could approach the problem?

What are some other words you can use to explain the problem?

What is your plan to solve the problem?

Does this make sense?

What can you do if you struggle?

How do you know if you are struggling?

How do you know when you have solved the problem?

Why is it important to keep trying to solve the problem?

How does making sense of problems and persevering in solving them help you?

Ownership Statements:

The problem is asking me to _____.

Being able to explain the problem is important because _____.

I can start to solve the problem by _____.

Making a plan to solve the problem is important because _____.

I solved the problem by _____.

I know when I have the answer because _____.

When I struggle, I can _____.

If I can't solve the problem yet, I try again by _____.

Trying many times to understand and solve the problem helps me
because _____.

Making sense of problems and persevering in solving them helps me _____.