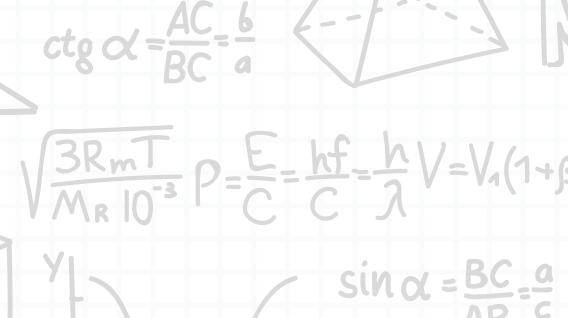


# Tools for Teaching the Practices

 $(X+C)(X-XC+C^{2})=X+C$ 

Pre-K through High School



elevated chievement

# The Standards for Mathematical Practice

At their core, the Standards for Mathematical Practice support mathematical metacognition. Metacognition is related to the concept of student ownership—a mindset that leads to elevated academic achievement. Students who own their learning are not thinking on a superficial level. They can state what they are learning and why, can explain how they learn best, can articulate when they are learning and when they are struggling, and understand their role in any academic setting. This is one type of "thinking about thinking" that leads to greater academic success.

#### THE STANDARDS FOR MATHEMATICAL PRACTICE



Make sense of problems and persevere in solving them.



Reason abstractly and quantitatively.



Construct viable arguments and critique the reasoning of others.



Model with mathematics.



Use appropriate tools strategically.



Attend to precision.



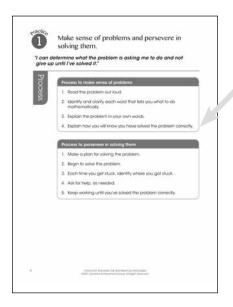
Look for and make use of structure.



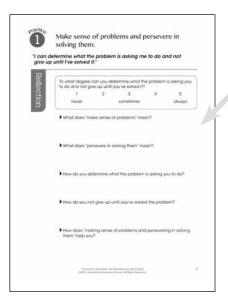
Look for and express regularity in repeated reasoning.

Fostering metacognition requires a balance of explicit instruction, teacher modeling, student-centered exploration, and responsive coaching that helps students first identify the thought processes they can apply, and then grow to use them on their own. Teachers, especially teachers of young children, will provide much more guidance, modeling, and support when teaching these processes, than teachers in the upper grades.

This guide provides tools for supporting students' metacognition as you teach the Standards for Mathematical Practice. For each standard you will find, a step-by-step process for learning the practice and a customized reflection that supports metacognition. On the back cover you will find the Student Ownership Statements that students will use when they are owning their learning of that practice.

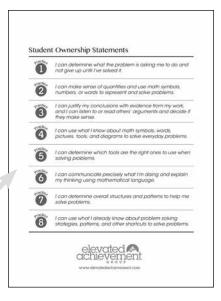


A step-by-step process for initial instruction, modeling, and guiding students to master the mathematical practice



Student Ownership
Statements that indicate
when students are
owning their learning
of the mathematical
mathematical practice

A customized reflection to support students as they "think about their thinking" and where they are in the mastery of the mathematical practice





# Make sense of problems and persevere in solving them.

"I can determine what the problem is asking me to do and not give up until I've solved it."

# Process

### Process to make sense of problems

- 1. Read the problem out loud.
- 2. Identify and clarify each word that tells you what to do mathematically.
- 3. Explain the problem in your own words.
- 4. Explain how you will know you have solved the problem correctly.

## Process to persevere in solving them

- 1. Make a plan for solving the problem.
- 2. Begin to solve the problem.
- 3. Each time you get stuck, identify where you got stuck.
- 4. Ask for help, as needed.
- 5. Keep working until you've solved the problem correctly.



# Make sense of problems and persevere in solving them.

"I can determine what the problem is asking me to do and not give up until I've solved it."

Reflection

To what degree can you determine what the problem is asking you to do and not give up until you've solved it?

1 2 3 4 5 never sometimes always

- ▶ What does "make sense of problems" mean?
- ▶ What does "persevere in solving them" mean?
- ▶ How do you determine what the problem is asking you to do?
- ▶ How do you not give up until you've solved the problem?
- How does "making sense of problems and persevering in solving them" help you?



# Reason abstractly and quantitatively.

"I can make sense of quantities and use math symbols, numbers, or words to represent and solve problems."

# Process

#### Process to reason abstractly

- 1. Clarify the context of the problem.
- 2. Explain how each number or symbol relates to the problem.
- 3. Translate the situation in the problem into an equation.
- 4. Sequence order of operations to solve the problem.

#### Process to reason quantitatively

- 1. Clarify context of the problem.
- 2. Identify quantities within the problem.
- 3. Sequence order of operations.
- 4. Use quantities and sequence to solve the problem.



# Reason abstractly and quantitatively.

"I can make sense of quantities and use math symbols, numbers, or words to represent and solve problems."

Reflection

To what degree can you make sense of quantities and use math symbols, numbers, or words to represent and solve problems?

1 2 3 4 5 never sometimes always

- ▶ What does "reason abstractly" mean?
- ▶ What does "reason quantitatively" mean?
- ▶ How do you make sense of quantities?
- ▶ How do you use math symbols, numbers, or words to represent and solve problems?
- ▶ How does reasoning abstractly and quantitatively help you?



# Construct viable arguments and critique the reasoning of others.

"I can justify my conclusions with evidence from my work, and I can listen to or read others' arguments and decide if they make sense."

**Process** 

#### Process to construct viable arguments

- 1. Explain what the problem is asking you to do with evidence.
- 2. Explain your plan to solve the problem and your reasons.
- 3. Explain your answer and why it is correct.
- 4. Explain your thinking in both writing and speaking.

#### Process to critique the reasoning of others

- 1. Ask clarifying questions as someone explains what the problem is asking them to do.
- 2. Ask clarifying questions as someone explains their plan to solve the problem.
- 3. Ask clarifying questions as someone explains their answer and how they solved the problem.
- 4. Cite similarities and differences between strategies for how the problem could be solved.
- 5. Locate the error in an incorrect answer and explain why with evidence.



# Construct viable arguments and critique the reasoning of others.

"I can justify my conclusions with evidence from my work, and I can listen to or read others' arguments and decide if they make sense."

Reflection

To what degree can you justify your conclusions with evidence from your work, and can you listen to or read other's arguments and decide if they make sense?

1 2 3 4 5 never sometimes always

- ▶ What does "construct viable arguments" mean?
- ▶ What does "critique the reasoning of others" mean?
- ▶ How do you justify your conclusion with evidence from your work?
- ▶ How do you listen to and read others' arguments and decide if they make sense?
- ▶ How does constructing viable arguments and critiquing the reasoning of others help you?





# Model with mathematics.

"I can use what I know about math symbols, words, pictures, tools, and diagrams to solve everyday problems."

## Process to model with mathematics

- 1. Clarify the problem and estimate a solution.
- 2. Identify which math symbols, words, pictures, tools, and diagrams will help you represent the problem mathematically.
- 3. Represent the problem mathematically.
- 4. Solve the problem accurately.
- 5. Determine if the results make sense and revise the model as needed.



# Model with mathematics.

"I can use what I know about math symbols, words, pictures, tools, and diagrams to solve everyday problems."

Reflection

To what degree can you use what you know about math symbols, words, pictures, tools, and diagrams to solve everyday problems?

1 2 3 4 5

never sometimes always

▶ What does "model with mathematics" mean?

▶ How do you use what you know about math symbols, words, pictures, tools, and diagrams to solve everyday problems?

▶ How does modeling with mathematics help you?



# Use appropriate tools strategically.

# "I can determine which tools are the right ones to use when solving problems."

Process

## Process to use appropriate tools strategically

- 1. Clarify the problem and estimate a solution.
- 2. Identify which tools you could use to find an exact answer.
- 3. Choose the best tool to use and explain your reasoning.
- 4. Solve the problem.
- 5. Identify if there were other tools you could have used that would have worked better.



# Use appropriate tools strategically.

# "I can determine which tools are the right ones to use when solving problems."

Reflection

To what degree can you determine which tools are the right ones to use when solving problems?

1 2 3 4 5

never sometimes always

▶ What does "use appropriate tools strategically" mean?

▶ How do you determine which tools are the right ones to use when solving problems?

▶ How does using appropriate tools strategically help you?



# Attend to precision.

# "I can communicate precisely what I'm doing and explain my thinking using mathematical language."

# **Process**

### Process to attend to precision

- 1. Explain the problem using specific mathematical language for words and symbols.
- 2. Explain the problem using specific units of measure.
- 3. Explain how you will solve the problem using specific mathematical language and units of measure.
- 4. As you are solving the problem, explain what you are doing using specific mathematical language and units of measure.
- 5. Explain your thinking in both writing and speaking.



# Attend to precision.

# "I can communicate precisely what I'm doing and explain my thinking using mathematical language."

Reflection

To what degree can you communicate precisely what you are doing and explain your thinking using mathematical language?

1 2 3 4 5

never sometimes always

▶ What does "attend to precision" mean?

▶ How do you communicate precisely what you are doing and explain your thinking using mathematical language?

How does attending to precision help you?



# Look for and make use of structure.

"I can determine overall structures and patterns to help me solve problems."

Process

#### Process to look for and make use of structure

- 1. Identify all parts of the problem.
- 2. Identify which parts go together and determine the structures and patterns.
- 3. Describe the structures and patterns in words and symbols.
- 4. Explain how determining the structures and patterns helped you solve the problem.



# Look for and make use of structure.

# "I can determine overall structures and patterns to help me solve problems."

Reflection

To what degree can you determine overall structures and patterns to help you solve problems?

1 2 3 4 5

never sometimes always

What does "look for and make use of structure" mean?

▶ How do you determine overall structures and patterns to help you solve problems?

▶ How does looking for and making use of structure help you?



# Look for and express regularity in repeated reasoning.

"I can use what I already know about problem solving strategies, patterns, and other shortcuts to solve problems."

### Process to look for and express regularity in repeated reasoning

- 1. Identify the type of problem.
- 2. Identify what operations within this problem you already know how to do.
- 3. Identify what you do over and over again when solving this type of problem.
- 4. Identify which strategies, rules, or patterns you could use to solve this problem.
- 5. Identify other shortcuts you could use based on the patterns you noticed.
- 6. Explain how you can repeat this thinking when solving similar problems.



# Look for and express regularity in repeated reasoning.

"I can use what I already know about problem solving strategies, patterns, and other shortcuts to solve problems."

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To what degree can you use what you already know about problem solving strategies, patterns, and other shortcuts to solve problems?

1 2 3 4 5

never sometimes always

▶ What does "look for and express regularity in repeated reasoning" mean?

▶ How do you use what you already know about problem solving strategies, patterns, and other shortcuts to solve problems?

▶ How does looking for and expressing regularity in repeated reasoning help you?

# **Student Ownership Statements**



I can determine what the problem is asking me to do and not give up until I've solved it.



I can make sense of quantities and use math symbols, numbers, or words to represent and solve problems.



I can justify my conclusions with evidence from my work, and I can listen to or read others' arguments and decide if they make sense.



I can use what I know about math symbols, words, pictures, tools, and diagrams to solve everyday problems.



I can determine which tools are the right ones to use when solving problems.



I can communicate precisely what I'm doing and explain my thinking using mathematical language.



I can determine overall structures and patterns to help me solve problems.



I can use what I already know about problem solving strategies, patterns, and other shortcuts to solve problems.

